Journal of International Business and Economics
September 2014, Vol. 2, No. 3, pp. 125-147
ISSN: 2374-2208 (Print), 2374-2194 (Online)

Published by American Research Institute for Policy Development
DOI: 10.15640/jibe.v2n3a6
URL: http://dx.doi.org/10.15640/jibe.v2n3a6

Aggregate Production Function, Its Determinants and Their Impacts on Aggregate Output: Case of Bahrain

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Abstract

In this paper an attempt is made to examine and analyse the influence of a few determinants of the level of real output (RGDP) of Kingdom of Bahrain, using Aggregate Cobb-Douglas Production Function (APF). Besides the inputs of labor (LF) and capital accumulation (GCF), two other factors believed to have essential impacts on the level of aggregate output, namely expenditure on educating (EDUR) and life expectancy (LIFEXP) are added to the traditional Cobb-Douglas production function as determinants of real output. Time series data on all the relevant variables have been used and tested for stationarity and association among them, using Augmented Dickey-Fuller test of unit root and Co-integration test. The null hypothesis of unit root in Augmented Dickey-Fuller test is rejected and we conclude that the data are stationary at first difference. Co-integration test indicates only long-run associations amongst the variables in the analysis. VAR technique is used to examine the variability of the RGDP of Bahrain. Important finding of this analysis is that all inputs – capital accumulation (GCF), labor force LF), expenditures on education (EDUR) and life expectancy - have significant impact on the level of GDP, but in varying degrees. LF has the most powerful influence while GCF has the least as suggested by the elasticity of output with respect to labor force and capital formation. Moreover, increasing return to scale is indicated.

Keywords: Kingdon of Bahrain, Real Gross Domestic Product (RGDP) of Bahrain, Aggregate Production Function (APF), labor force of bahrain(LF), Fixed Gross Capital Formation (GCF), Education(EDUR), Life expectation(LIFEXP)

I. Introduction

Bahrain is among the high-income countries in the world with RGDP per-capita estimated at US $ 18,461 (2005).

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With its highly developed communication and transport facilities, Bahrain is now home to numerous multinational firms with business in the Arabian Gulf.

During the 1970s and early 1980s, Bahrain experienced very high levels of growth. Income from oil accounted for approximately 80 percent of Government revenues during 1974-75 alone. The collapse of oil prices in the 1980s resulted in considerably slower economic growth and development in the region. Endowed with smaller oil resources than its Neighbors, Bahrain has established one of the most diversified economies in the region. In 2005, the services sector, led by financial services[2], contributed 74.3% to Bahrain's real GDP, and employed over 50% of the workforce; manufacturing, developed on the basis of Bahrain's comparative advantages in energy-intensive industries mainly aluminum, was responsible for 13.3% of real GDP; mining and quarrying sector, notably oil and gas, accounted for 11.8% of real GDP; while agriculture accounted for only 0.6% of real GDP.

The services sector is a key component in Bahrain's overall policy of economic diversification; financial services, in particular, have developed strongly over the last few years. In July 2004, Bahrain liberalized its fixed and international telecommunication services. Private sector participation is being encouraged though the removal of obstacles to foreign investment. Nevertheless, provision of certain services, such as road and maritime transport, is restricted to Bahraini nationals.

The mining and quarrying sector, basically petroleum and natural gas, is dominated by several state-owned companies, notably Bahrain Petroleum Company (BAPCO) and Bahrain National Gas Company (BANAGAS). The Government is seeking to further develop its petroleum resources either directly or in cooperation with foreign enterprises through production sharing agreements. Bahrain is also increasing its electricity network and upgrading generation capacity in order to meet its growing demand.

Bahrain's manufacturing sector is based on its comparative advantage in energy-intensive industries, particularly aluminum. Despite recent privatizations, the State retains a significant role in the sector.

[2] Bahrain's financial services subsector has developed strongly over the last few years, becoming one of the main engines of economic growth. It serves both the domestic economy and those of its neighbours.

The Government holds a majority stake or is an important shareholder in manufacturing companies, such as Aluminum Bahrain (ALBA), and the Arab Shipbuilding and Repair Yard Company (ASRY). Manufacturing is being promoted partly through investment incentives, including exemption from import duties, and a five-year grace period on rent payment for land in the industrial zones. MFN tariffs on manufactured imports average 5.4% (down from 8% in 2000), ranging up to 125% on alcoholic beverages.

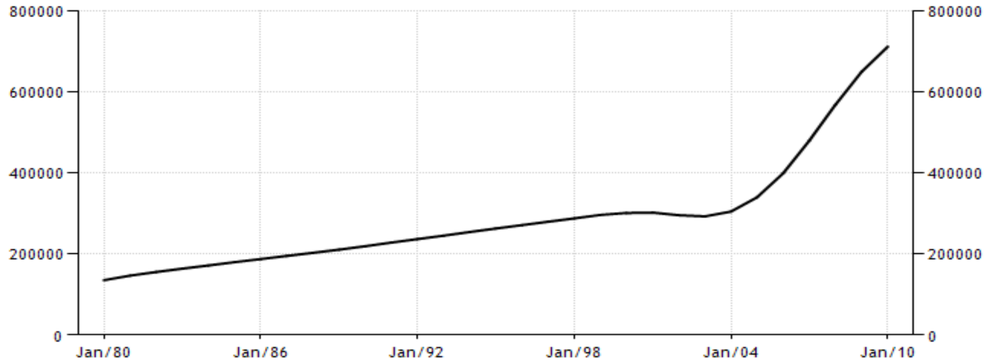
Bahrain has decided to go ahead with its key government backed development projects to enhance its economic indicators, such as the RGDP in its draft 2009-10 budget. During 2008, Bahrain stepped up efforts to bolster the country's infrastructure, launching an upgrade of the existing port and expanding the Bahrain International Airport to increase its cargo handling capacity annually. However, such public projects may be significantly constrained by falling oil prices in future.

There is no doubt that the world of work is changing. Rapidly expanding emerging economies, an increasingly global labor market and new technology are all having a significant impact on the level of out output and economic growth. Economic growth can be increased by increasing the amount and types of labor and capital used in production, and by attaining greater overall efficiency in how these factors of production are used together.

The most prominent feature of Bahrain’s labor market is its segmentation between national and expatriate workers. Nationals accounted for 90 per cent of the public sector work force in 2000, while two thirds of workers in the private sector were non-nationals. Foreigners make up nearly 60 per cent of the work force, a proportion that has changed little over the past 15-20 years. Most work permits issued to expatriates are for unskilled and semi-skilled workers. In 2000, for example, professional, technical, administrative and managerial workers accounted for just 10 per cent of the total.

The Labor force; total in Bahrain was last reported at 711371.09 in 2010, according to a World Bank report published in 2012. Total labor force comprises people ages 15 and older who meet the International Labor Organization definition of the economically active population: all people who supply labor for the production of goods and services during a specified period.

It includes both the employed and the unemployed. While national practices vary in the treatment of such groups as the armed forces and seasonal or part-time workers, in general the labor force includes the armed forces, the unemployed, and first-time job-seekers, but excludes homemakers and other unpaid caregivers and workers in the informal sector. This following chart, indicate the growing labor force of bahrain.



The Human Development Index (HDI) value for Bahrain rose from 0.820 in 2000 to 0.895 in 2009. According to Human Development Report, Bahrain ranked 39th globally. Sustainable human development in a country is measured through progress in four key areas, namely, (i) life expectancy at birth; (ii) literacy rate of population ages 15 and above; (iii) combined secondary and higher education enrolment ratio; and, (iv) RGDP per-capita in terms of purchasing power parity.

I. Theoretic Model

The Cobb-Douglas pf is used in microeconomics to study relationships between output and its 2 traditional inputs, labor and capital (1900-1926). Similar functions were originally used by Knut Wicksell (1851-1926). However, the use of such functions in estimating aggregate output of an economy is limited because of theoretical problems it brings in aggregation. Still, with all its problems, the estimation and use of aggregate production function has become a wide-spread practice in macroeconomic analysis (F.M.Fisher 1969).

Arguments, both for and against the existence of the aggregate production function, linger in the literature for a long time (Simon and Levy 1963; Simon, 1979; Shaikh 1980; Felipe 2001; Felipe and McCombie 2001-2003 & 2005- 2006). Such arguments can be resolved by either of the following ways:

In the economics of joint production, one can often distinguish between a case where a firm produces multiple products, each under separate function process, and the case where a number of outputs are produced from a single production process with common inputs. In the econometric practice the first case has often been dealt with by aggregation of individual production function into a macro production function. The second case has often been used for estimation of an implicit aggregate production function.

Cobb-Douglas "Aggregate" Production Function (Apf)

The aggregate production function is the maximum output that can be produced given the quantities of the factors of production. The starting point for analysis of the Classical engine is the production function:

$$Y = f(K^*, L) \qquad (1)$$

The Classical production function shows different levels of output (Y) assuming fixed technology and varying amounts of factors of production (K^* = capital in the form of plant and equipment; L = labor measured in homogeneous units). In the short-run it is assumed that the amount of capital is fixed (indicated by the symbol over K, but in the long run capital is varied too) and varying quantities of labor but assuming a fixed population (otherwise additional labor would become available simply through natural growth). This function displays the following three properties: the function (1) is increasing (possibly weakly), (2) displays constant returns to scale, (3) and displays diminishing returns.

In the macroeconomic environment, the theoretical analysis of economic growth is the nature of the relationship between an economy's factors of production and its output. Assuming homogeneity of the aggregate production function, an important element of this relationship is the degree of returns to scale.

In neoclassical growth models, constant returns to scale is usually assumed to prevail due to perfect competition. In contrast, endogenous growth models typically assume increasing returns to scale as a result of technological progress that is usually modeled as arising from the accumulation of physical or human capital (e.g., Romer (1987) and Lucas (1988). For production, the function is

$$Y = AL^{\alpha} K^{\beta} \qquad (2)$$

Where:

Y = total production (the monetary value of all goods produced in a year)
L = labor input
K = capital input
A = total factor productivity
α and β are the output elasticities of labor and capital, respectively. These values are constants determined by available technology.

The non-linear Cobb-Douglas pf (2) above, can be estimated in linear form as

Ln Y = ln L + ln K ; (2'), where ln = $\log_e$

With many factors, the Cobb–Douglas production function can generally be estimated as a linear relationship using the following expression:

$$ln(Y) = a_o + \sum a_i \, ln(I_i) \qquad (3)$$

Where:
Y = Output
I_i = Inputs
a_i = model coefficients

We are extending the pf to include human capital which consists of 2 components, health (proxy: Life Expectancy, LIFEXP) and public expenditure on education, EDUR. We assume that the effect of health and education on output depends only on the average level of health and education in the economy and not on its distribution. For policy purposes, we estimate the effect of increasing health on average. Particular health interventions that affect different sections of the society in different ways may have a greater or lesser effect than this.

Education is well-known to have a direct and positive effect on economic growth and productivity. Many studies have found that some channels through which education affects growth are productivity and development in general (Denison 1967,1979; Jamison & lau 1982; Lou & Yotopoulos 1989; Hayami & Ruttan 1985; Human Capital Approach, pioneered by Schultz 1961 & Psacharopoulos 1985). Moreover, there is evidence from the experience of many countriesthat education, by enabling the acquisition of the necessary skills by the workers, is in fact a complementary inputto physical capital and technology. Having physical and financial capital as well as access to technology is not enough; there must be skilled manpower to make use of these resources*.

Although labor quality, in the form of human capital, clearly contributes significantly contributes to economic growth, most empirical studies identify human capital narrowly with education.

Health is a crucial aspect of human capital and therefore is a critical ingredient of economic growth. Healthier workers are physically and mentally more energetic and robust. They are more productive and earn higher wages. A substantial body of microeconomic evidence indicates such effects (Strauss & Thomas 1998). Therefore, this factor is included in the model to examine whether this micro evidence can be corroborated by macro evidence of an effect of population health on economic growth. Health, in the form of life expectancy, has appeared in many cross-country growth regressions, and investigators generally found that it has a significant positive effect on the rate of economic growth (Bloom & Canning 2000, 2003)

*For example, the successes of South Korea and Taiwan in developing their respective economies and the failure of Thailand, until recently, to develop hers, may be particularly attributed to relatively lower level of educational development in Thailand in the 1960s and 1970s.By late 1980s, however, Thailand has finally caught up with the level of educational development achieved by South Korea and Taiwan in the early 1960s and is now well on her way to becoming the 5th “newly industrialized economy” (Lau,et.al, 1991).

We model our aggregate output as a function of inputs and technologies using the following aggregate production function:

$$Y = A(GCF) \quad (LF)^{\beta} \; (LIFEXP) \; (EDUR) \qquad (4)$$

Where,

Y is real output or real gross domestic product (RGDP)
Total Factor Productivity (TFP) is A
Gross Capital Formation is GCF
Labor force is LF
Health (which we proxy with life expectancy) is LIFEXP
Public expenditure of education is EDUR.
B_i (i=2,….., 5) indicates output elasticities of production inputs, respectively. Sum of these elasticities indicate the nature of returns-to-scale.

We estimate the effect of increasing health on average; particular health interventions that affect different sections of society in different ways may have a greater or lesser effect than this.

Taking logs of the aggregate production function in (4) above, we derive an equation for the log of Output in time t:

$$\ln Y = \beta_1 + \beta_2 \ln GCF + \beta_3 \ln LF + \beta_4 \ln EDUR + \beta_5 \ln LIFEXP \quad (5)$$

II. Data, Methodology & Empirical Tests

Data for all the variables used in this paper is from the Word Development Indicators WID for the period 1985-2010 and Statistical Abstract of Kingdom of Bahrain 1999.

Data used in any econometric model have to be tested Non-stationarity. Why do we need to test for Non-Stationarity? Often, ordinary least squares (OLS) is used to estimate the slope coefficients of the autoregressive model. Use of OLS relies on the stochastic process being stationary. When the stochastic process is non-stationary, the use of OLS can produce invalid estimates. Granger and Newbold (1974) called such estimates 'spurious regression' results: high R^2 values and high t-ratios yielding results with no economic meaning.

Augmented Dickey-Fuller Test

In statistics, a **unit root test** tests whether a time series variable is non-stationary is used. The most famous test is the Augmented Dickey–Fuller test.

Another test is the Phillips–Perron test. Both these tests use the existence of a unit root as the null hypothesis. To estimate the slope coefficients, we can;

- assume the process is stationary (has no unit roots) and use OLS, or
- Assume that the process has a unit root, and apply the difference operator to the series. OLS can then be applied to the resulting (stationary) series to estimate the remaining slope coefficients.

When data are non-stationary undesired outcomes can be deducted:

- The stationarity or otherwise of a series can strongly influence its behavior and properties e.g. persistence of shocks will be infinite for non-stationary series
- Spurious regressions. If two variables are trending over time, a regression of one on the other could have a high *R*2 even if the two are totally unrelated
- If the variables in the regression model are not stationary, then it can be proved that the standard assumptions for asymptotic analysis will not be valid. In other words, the usual "*t*-ratios" will not follow a *t*-distribution, so we cannot validly undertake hypothesis tests about the regression parameters.

However, non-stationary variables may be used in the regression if they prove to be co-integrated. There are three approaches to the problem of spurious regression. The first approach is to difference the data before estimating. The second approach is to add the lags of the dependent variable. Finally, one may consider using the co-integration technique.

In statistics and econometrics, an **Augmented Dickey–Fuller test (ADF)** is a test for a unit root in a time series samples. It is an augmented version of the Dickey–Fuller test for a larger and more complicated set of time series models. The augmented Dickey–Fuller (ADF) statistic, used in the test, is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit roots at some level of confidence.

The testing procedure for the ADF test is the same as for the Dickey–Fuller test but it is applied to the model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \dots + \delta_p \Delta y_{t-p} + \varepsilon_t \quad (6)$$

Where α is a constant, β the coefficient on a time trend and *p* the lag order of the autoregressive process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modeling a random walk and using the constraint $\beta = 0$ corresponds to modeling a random walk with a drift. Consequently, there are three main versions of the test, analogous to the ones discussed on the Wikipedia page for the Dickey-Fuller test. See that page for a discussion on dealing with uncertainty about including the intercept and deterministic time trend terms in the test equation.

By including lags of the order *p* (Greek for 'rho') the ADF formulation allows for higher-order autoregressive processes. This means that the lag length *p* has to be determined when applying the test. One possible approach is to test down from high orders and examine the t-values on coefficients. An alternative approach is to examine information criteria such as the Akaike information criterion, Bayesian information criterion or the Hannan-Quinn information criterion.

The unit root test is then carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$. Once a value for the test statistic $DF_\tau = \hat{\gamma}/SE(\hat{\gamma})$ is computed it can be compared to the relevant critical value for the Dickey–Fuller Test. If the test statistic is less (this test is non symmetrical so we do not consider an absolute value) than (a larger negative) the critical value, then the null hypothesis of γ equals 0 is rejected and no unit root is present.

Data of the model have been tested for the presence of unit root. The order of integration for each variable is determined using Augmented Dickey and Fuller (ADF).

Table 1: Augmented Dickey and Fuller (ADF) results

Variable	Constant	Constant and Trend	No Constant & No Trend
Log Level			
GDP	1.512438*	-4.187269**	4.210184
GCF	-2.847097***	-2.795717	0.010574
EDU	-0.907019	-0.684128	-2.272120**
Life-Exp.	-4.377699*	-2.241464	7.135603
LF	-3.945703*	-2.762839	1.125484
Log First Difference			
GDP	-4.102311*	-4.34970**	-2.399985*
GCF	-5.797241*	-5.791444*	-5.929008*
EDU	-7.392807*	-7.331152*	-2.366345**
Life-Exp.	-3.753402*	-4.975804*	-2.030484**
LF	-1.691456	-3.841241**	-2.478209**

*Reject Null Hypothesis (unit root) at 1%
**Reject Null Hypothesis (unit root) at 5%

Tables.1 shows the empirical and stationarity tests results which indicate that the variables are non stationary in levels, however, with first difference they become stationary, thus they are I(1). Co-integration test are necessary to be examined to reflect the long run and stable relationship between the variable exist. The results of the unit root and test are reported in table 1 suggest that all the variables contain a unit root. Since the five variables are noted to be I(1), there exist the possibility that they share a long run equilibrium by using cointegration test as stated by Engle and Granger (1987).

Co-Integration Test

Empirical research in macroeconomics as well as in financial economics is largely based on time series. Ever since Economist Laureate Trygve Haavelmo's work it has been standard to view economic time series as realizations of stochastic processes. This approach allows the model builder to use statistical inference in constructing and testing equations that characterize relationships between economic variables.

Some applied studies including a study of (Nelson and Plosser (1982) and study of Phillips (1987) stated that much of the time series are unstable because they contain a unit root, where the presence of unit root in any time series will lead to correlation between the average and the variance of the variable and time; Granger; Newbold (1974). Therefore, there will be time series analysis of the variables under study to test the stability of time series over time and determine the degree of integration by testing the relationship equilibrium, and test the causal relationship in the short and long-terms.

Before testing the existence of the relationship between long-term real GDP (RGDP), and other variable in the model, and the analyze of the behavior of the relationship in the short term. Time series should be test to confirm their stability over time and determine the degree of integration. If an OLS regression is estimated with non-stationary data and residuals, then the regression is spurious.

Cointegration is a statistical property possessed by some time series data that is defined by the concepts of stationarity and the order of integration of the series. A stationary series is one with a mean value which will not vary with the sampling period. For instance, the mean of a subset of a series does not differ significantly from the mean of any other subset of the same series. Further, the series will constantly return to its mean value as fluctuations occur. In contrast, a non-stationary series will exhibit a time varying mean. The order of integration of a series is given by the number of times the series must be differenced in order to produce a stationary series. A series generated by the first difference is integrated of order 1 denoted as I(1). Thus, if a time series, is I(0), it is stationary, if it is I(1) then its change is stationary and its level is non-stationary. To overcome this problem the data has to be tested for a unit roots (i.e. whether it is stationary).

Co-integration is said to exist between two or more non-stationary time series if they possess the same order of integration and a linear combination (weighted average) of these series is stationary. Thus, if x_t and y_t are non-stationary and are of the same order, there may exist a number **b** such that, the residual series, g_t, $(= y_t - bx_t)$ is stationary. In this case x_t and y_t are said to be cointegrated with a cointegrating factor of b.

The significance of Cointegration analysis is its intuitive appeal for dealing with difficulties that arise when using non-stationary series, particularly those that are assumed to have a long-run equilibrium relationship. For instance, when non-stationary series are used in regression analysis, one as a dependent variable and the other as an independent variable, statistical inference becomes problematic [Granger and Newbold, 1974]. Cointegration analysis has also become important for the estimation of error correction models (ECM). The concept of error correction refers to the adjustment process between short-run disequilibrium and a desired long run position. As Engle and Granger (1987) have shown, if two variables are cointegrated, then there exists an error correction data generating mechanism, and vice versa.

Since, two variables that are cointegrated, would on average, not drift apart over time, this concept provides insight into the long-run relationship between the two variables and testing for the Cointegration between two variables such as RGDP and labor force would also be a test of the validity of an error correction specification involving these variables. With regard to testing procedures for the error correction model, once Cointegration is ascertained, then the residuals from the cointegrating test, lagged one period, are used in a vector autoregression involving the appropriate differencing of the series (to ensure stationarity) forming the hypothesized relationship. The empirical results of these relationships are presented in the following tables. The trace and Max eigen-value statistic for testing the rank of Cointegration are shown in Tables 2 and 3, respectively. The results of both tests deny the absence of cointegrating relation between the variables series. Furthermore, both tests suggest the presence of one cointegrating equation at 5% level.

Table 2: Co integration Results Unrestricted Co Integration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	Critical Value 0.05	Prob.**
None *	0.890024	137.791	88.8038	0.0000
At most 1 *	0.780627	84.8109	63.8761	0.0003
At most 2 *	0.659501	48.4034	42.9153	0.0129
At most 3	0.436358	22.5472	25.8721	0.1228
At most 4	0.306586	8.7871	12.5179	0.1939

Trace test indicates 3 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 3: Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	Critical Value0.05	Prob.**
None *	0.890024	52.97991	38.331	0.0006
At most 1 *	0.780627	36.40752	32.118	0.0140
At most 2 *	0.659501	25.85624	25.823	0.0495
At most 3	0.436358	13.76008	19.387	0.2706
At most 4	0.306586	8.787088	12.518	0.1939

Max-eigenvalue test indicates 3 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Cointegration tests are presented in Tables 4 and 5, respectively. These results suggest that a long run and stable relationship between the variables exists. Further, the results indicate that real gross domestic product (GDP) has a long run significant impact on gross capital formation (GCF).

Table 4: Unrestricted Cointegrating Coefficients (Normalized by b'*S11*b=I)

LGDPF1	LGCF	LEDUR	LLF	LLIFEXP	@TREND(81)
-13.92	-0.42	-0.735	-24	188.78	0.959888
40.973	-1.6	0.0228	138	-628.3	-3.880617
-2.012	-0.98	17.98	-121	327.89	2.198285
4.2204	5.45	-7.127	112	-258.5	-2.683937
-5.406	0.88	-5.57	-78	445.11	1.400675

Table 5: Unrestricted Adjustment Coefficients (alpha)

D(LGDPF1)	0.0319	-0.0095	0.0086	0.0023	-0.0102
D(LGCF)	0.1086	0.0418	-0.0567	-0.1769	-0.0139
D(LEDUR)	0.012	-0.0416	-0.0501	-0.0125	0.0121
D(LLF)	-0.005	-0.001	-0.0001	-0.0008	-0.0003
D(LLIFEXP)	-0.0012	-1.87E-05	-0.00052	0.00019	-0.000929

Error Correction Model

We have rejected the null hypothesis of no cointegration. The Error Correction model ECM is then formed using the residual lagged one time period as the error correction term.

The ECM model is used to examine the short run analysis. As with short-run models including lags, it can be used for forecasting. The coefficient on the error correction term can be used as a further test for cointegration.

The error correction term tells us the speed with which our model returns to equilibrium following an exogenous shock. It should be negatively signed, indicating a move back towards equilibrium; a positive sign indicates movement away from equilibrium. The coefficient should lie between 0 and 1, 0 suggesting no adjustment one time period later, 1 indicates full adjustment The following ECM was formed and the following results were found:

$$\Delta GDP = 0.119 - 1.944\Delta LF - 0.814LIFEXP + 0.0405GCF - 0.152EDUR - 0.457ECT(-1)$$
$$(0.030381)\ (1.0209)\quad (3.2395)\quad (0.0239)\quad (0.0900)\quad (0.1317)$$
$$R^2 = 0.54 \qquad DW = 1.94$$
$$SE\ is\ in\ parenthese$$

The error correction term has a t-statistic of -3.47, which is highly significant (P=0.0026) supporting the cointegration result. The coefficient on the error correction term is negative, so the model is stable. The coefficient of -0.457, suggests 45.7% movement back towards equilibrium following a shock to the model, one time period later.

Variance Decompositions

The Cointegration analysis so far only suggests the long-run associations amongst variables in the analysis. However, our objective is also to examine the relative strength of each variable in explaining the changes in the dependent variable. Here, we implement an unrestricted VAR model. From the model, we generate variance decompositions (VDCs) and Impulse Response functions (IRFs) to capture the relative importance of various shocks and their influences on our variable of interest. The orderings that we have chosen are: DGDP, DGCF, DLIFEXP, DLF, and DEDUR. This is based on the assumption that GDP influence DGCF, DLIFEXP, DLF, and DEDUR.

Tables 6, 7, 8, 9, and 10 show the variances decomposition of the GDP over 6 periods. The statistics indicate the percentage contribution of innovations in each of the variables in the system to the variance of the GDP. The results show that shocks to the GDP itself, Life Expectation LIFEXP, the Labor Force LF, and Education EDUR over all horizons. Not much can be attributed to Gross Capital Formation GCF although over longer horizons its relative contribution increases.

More importantly, the variance decomposition of the Gross Capital Formation GCF (Tables 6 to 10) shows that apart from innovations to GCF itself, GDP contributes significantly to the variations in the GCF. We can conclude that the basic transmission mechanism runs from base GDP to Gross Capital Formation. Further the contribution of innovations in the Life Expectation LIFEXP suggests that much variation in LIFEXP will enhance GCF, and this supported by (Strauss & Thomas, 1998). Fewer contributions were reflected by LF and EDUR.

Table 6 :Variance Decomposition of DGDP

Period	S.E.	DGDP	DGCF	DLIFEXP	DLF	DEDUR
1	0.036	100.0	0.000	0.000	0.000	0.000
2	0.048	63.65	0.513	26.09	6.855	2.897
3	0.050	58.43	0.999	30.37	6.953	3.252
4	0.056	58.71	0.851	30.85	5.944	3.644
5	0.059	56.72	0.837	32.53	6.111	3.809
6	0.063	54.48	0.917	34.45	6.083	4.071

Table 7: Variance Decomposition of DGCF

Period	S.E.	DGDP	DGCF	DLIFEXP	DLF	DEDUR
1	0.464	8.959	91.04	0.000	0.000	0.000
2	0.541	6.640	85.52	6.498	0.634	0.711
3	0.675	12.48	81.63	4.829	0.563	0.504
4	0.739	10.93	82.68	5.317	0.483	0.592
5	0.813	10.98	82.84	5.112	0.506	0.562
6	0.878	10.65	83.06	5.229	0.488	0.573

Table 8: Variance Decomposition of DLIFEXP

Period	S.E.	DGDP	DGCF	DLIFEXP	DLF	DEDUR
1	0.003	11.11	0.001	88.89	0.000	0.000
2	0.003	8.820	3.004	87.81	0.293	0.073
3	0.004	11.87	2.254	85.41	0.217	0.256
4	0.004	10.24	2.853	86.21	0.454	0.226
5	0.005	9.645	2.655	86.99	0.435	0.278
6	0.005	9.228	2.754	87.31	0.437	0.274

Table 9: Variance Decomposition of DLF

Period	S.E.	DGDP	DGCF	DLIFEXP	DLF	DEDUR
1	0.004	4.993	4.439	33.11	57.45	0.000
2	0.005	4.398	2.390	29.19	62.18	1.837
3	0.006	4.142	1.737	23.65	69.16	1.311
4	0.007	4.456	1.493	23.86	68.79	1.396
5	0.008	3.954	1.233	23.19	70.39	1.238
6	0.009	3.831	1.097	22.69	71.19	1.194

Table 10: Variance Decomposition of DEDUR

Period	S.E.	DGDP	DGCF	DLIFEXP	DLF	DEDUR
1	0.104	33.42	0.124	0.815	6.019	59.62
2	0.160	37.82	13.30	7.711	11.28	29.90
3	0.191	42.75	9.487	6.623	10.64	30.51
4	0.219	43.96	10.78	6.784	10.73	27.75
5	0.241	44.74	9.588	6.812	11.02	27.84
6	0.264	45.80	9.663	6.771	11.05	26.71

The variance decomposition of the Life Expectation is shown in the table. The figures indicate that a shock to the LIFEXP itself is highly significant. Furthermore contribution is noted to shocks of GDP, this comes from the fact that an increase in the level of national income will improve LIFEXP. But less contribution is found to the shock of GCF.

More importantly, the variance decomposition of the Labor Force LF shows that apart from innovations to the LF itself, Life Expectation contribute significantly to the variations in the Labor force, and more less to GDP.

Finally, the table shows the variance decomposition of the Education EDUR. The greater contribution of innovations in the Education suggests that much of its volatility is the result of GDP (even in the long run). One will also note the increasing contribution of the LF over time.

Granger Casualty Test

To test the existence of a long-run relationship between each two variables, we also implemented the Granger Causality test within an error-correction framework. To analyze the relationship between, causality among these variables using the method developed by Granger (1969). Granger causality test is one of the most interesting and widely used VAR applications. The intuition behind it is simple: If previous values of variable X significantly influence current values of variable Y, then one can say that X causes Y. Since this technique is used in a number of economic studies, only brief explanations of these method is provided below.

A general specification of the Granger causality test in a bivariat (X, Y) context can be expressed as follows:

$$Y_t = \alpha_o + \alpha_1 Y_{t-1} + \cdots + \alpha_i Y_{t-i} + \beta_1 X_{t-1} + \cdots + \beta_i X_{t-i} + \varepsilon \quad (7)$$

$$X_t = \alpha_o + \alpha_1 X_{t-1} + \cdots + \alpha_i X_{t-i} + \beta_1 Y_{t-1} + \cdots + \beta_i Y_{t-i} + \varepsilon \quad (8)$$

The existence of a long-run relationship between two variables for example GDP and GCF means that both variables are causally related at least one direction. But, whether change in variable is causing change in the second variable is still unknown. In order to learn the direction, we implemented the Granger causality test. We can obtain two tests from this analysis: the first examines the null hypothesis that the GDP does not Granger-cause GCF, and the second test examines the null hypothesis that the GCF does not Granger-cause the GDP. If we fail to reject the former null hypothesis and reject the latter, then we conclude that GDP changes are Granger-caused by a change in GCF. Therefore this test involves the examination of the statistical significance of the parameters of X in Eq. (7) and those of Y in Eq. (8).

To verify the existence of a long-run relationship between each two variables, F-statistics and probability values are constructed under the null hypothesis of noncausality in Table (11). It can be observed that there is a causal relationship between GDP and EDUR. However, our results show that one-way causality exists only from GDP to EDUR. The important finding is that causal relationship between GDP and LF is indicated, in one way direction from GDP to LF. Regarding GDP and LIFEXP, causality has been found in one way direction from GDP to LIFEXP. Finally, it can be seen that there a causality relationship between LIFEXP and LF in one way direction.

Table 11: Pairwise Granger Causality Tests Lags: 1

Probability	F-Statistic	Obs.	Null Hypothesis:
0.75195	0.10256	24	DGCF does not Granger Cause DGDPF1
0.43715	0.62744	24	DGDP does not Granger Cause DGCF
0.84381	0.03979	24	DEDUR does not Granger Cause DGDPF1
0.10106	2.94129	24	DGDP does not Granger Cause DEDUR
0.33901	0.95730	24	DLF does not Granger Cause DGDPF1
0.06479	3.79818	24	DGDP does not Granger Cause DLF
0.99847	3.8E-06	24	DLIFEXP does not Granger Cause DGDPF1
0.02912	5.48466	24	DGDP does not Granger Cause DLIFEXP
0.92387	0.00935	24	DEDUR does not Granger Cause DGCF
0.12791	2.51230	24	DGCF does not Granger Cause DEDUR
0.67846	0.17674	24	DLF does not Granger Cause DGCF
0.48309	0.50980	24	DGCF does not Granger Cause DLF
0.38861	0.77511	24	DLIFEXP does not Granger Cause DGCF
0.33106	0.99004	24	DGCF does not Granger Cause DLIFEXP
0.11365	2.72527	24	DLF does not Granger Cause DEDUR
0.71676	0.13522	24	DEDUR does not Granger Cause DLF
0.34556	0.93108	24	DLIFEXP does not Granger Cause DEDUR
0.99532	3.5E-05	24	DEDUR does not Granger Cause DLIFEXP
0.07007	3.64291	24	DLIFEXP does not Granger Cause DLF
0.23580	1.48961	24	DLF does not Granger Cause DLIFEXP

Elasticities and Return to Scale

The Cobb-Douglas production function is used to test the elasticity of output GDP with respect to capital and labor.

In this approach, the output elasticity with respect to each input must be estimated from production function using the share of each variable of the model. The logarithm production function stated in equation (5) is tested to include different variables starting from the full form i.e. Including all variable, eliminating some variables, and finally, to include only the two inputs Labor Force LF and Gross Capital Formation GCF as shown in table .12.

Table 12 :The Elasticity and Return to Scale Coefficients

Expl. Variable	Coeffi.	Model1 OLS	Model2 AR(2)	Model3 AR(2)	Model 4 AR(2)	Mode5 AR(2)
Cons	β_1	22.23	-10.5	-	-9.919	0.642
ln LF	β_2	2.9016^{***}	1.338^{***}	1.6887^{***}	1.2338^{*}	1.616^{***}
lnGCF	β_3	0.120^{***}	0.068^{**}	0.064^{***}	0.04907^{***}	0.0492^{*}
ln Lifexp	β_4	-8.59^{*}	3.460	-	3.4884	-
ln EDUR	β_5	-0.5^{**}	-0.13	-0.11	-	-
Adj. R^2		0.96	0.98	0.987	0.98	0.982
D-W		1.19	1.65	1.62	1.561	1.56

They indicate that the output elasticity of labor for the economy is greater than one and higher than capital elasticity, indicating that the real GDP is elastic with respect to LF. The elasticity of real GDP with respect to capital is less than one (inelastic). In other words, during the past two decades, the Bahrain economy relied more heavily on labor than capital in production processes. The important thing is that the coefficient of the inputs Labor force and Gross Capital Formation are mostly highly significant. Moreover, the adjusted R^2 is high in all models. Durbin-Watson statistics are all in line with high degree of model performance, except for model1.

The size of the return to scale in the aggregate production function has important implications for many questions in macroeconomics. Analyses of business cycle, growth rate, and the scope of government policy depend fundamentally on whether there constant return to scale (CRS), or increasing return to scale (IRS). For example, with IRS, indicates the importance of government policies (fiscal and monetary policies) to be used to improve economic welfare. Regarding economic growth, if aggregate production function technology is CRS and depend on measured inputs of capital and labor, then log-run growth could determined by technological factors, while model which exhibit IRS can imply that economic growth is largely due to increase accumulated factors. All tested models exhibit increasing return to scale (IRS). But, the more important factor is the labor force.

III. Summary Results & Remarks

The Aggregate Production Function APF was utilized in this paper as a device to identify the influence of labor, capital and other factors, such as LIFEXP and EDUR on the level of real output (RGDP) of Bahrain. Several approaches, e.g. VAR technique, were used for this purpose. Important among the findings of this analysis are as follows:

1. Both labor force and capital have significant impact on the level of RGDP; but less contribution comes from GCF. Other variables were either not so significant or irrelevant in case of Bahrain.
2. The contribution of innovations in all explanatory variables of the model came from the variability of the RGDP. This indicates the existence of mutual dependency between the dependent variable (RGDP) and the independent variables of the model. Any change in RGDP caused by the changes in independent variables produces a positive impact on improving the independent variables later.
3. Various output elasticities were also tested. Because of the Autoregressive problem, (AR) was added to the base model to test them. The results indicate that the elasticity of output (GDP) with respect of labor input is elastic; and the elasticity of output with respect of capital (GCF) was found highly significant, but, it is inelastic. Both are highly significant.
4. Adjusted R^2 is high in all models tested and Durbin-Watson d are all in line with high degree of model performance, except Model 1.
5. Aggregate Cobb-Douglas production function indicates that the output of Bahrain experiences an increasing return to scale (IRS), which imply that the rapid economic growth in Bahrain is fueled largely by increases in the accumulated factors of production, i.e. labor and capital. Life expectancy and real spending on education are less important due to the negative signs in the base model (OLS).

Some policy directions for RGDP growth for Bahrain may be suggested from our findings:

1. Growth priorities should be given to enhancing the labor force in order to achieve high performance of the Bahrain economy. This suggestion is not so surprising because Bahrain economy relies more heavily on labor than capital in production processes.
2. Any policy adopted by Bahrain government to enhance the level of real output will actually improve education and health of Bahraini people. So, the policy makers in Bahrain must aim at economic growth of Bahrain.

3. Capital formation appears to be a less significance source of growth of RGDP, although it is in most countries the only source to sustain high level of growth. Bahrain may be an exception now; but it must not ignore to form capital to sustain its growth. A big part of Bahrain labor force is foreign labor. If Bahrain ever runs problems of attracting them, its economic growth is likely to dissipate quickly. Bahrain must try to put its eggs in different baskets.

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